Problem bank for "Week 2" material: Quadratic functions, translation and reflection, exponential functions

08 Sep 2020

Group 1

Application to Public Health
The U.S. Center for Disease Control and Prevention projects that tobacco could soon be the leading cause of death in the world. In 1990, 35 million years of healthy life were lost globally due to tobacco. This quantity was rising linearly at a rate of about 28 million years each decade. In contrast, 100 million years of healthy life were lost due diarrhea with the rate falling linearly 22 million years each decade.

1. Write the years of healthy life in millions lost globally to tobacco as a linear functions $f(t)$ of the years t, since 1990.
2. Write the years of healthy life in millions lost globally to diarrhea as a linear function $g(t)$ of the years t, since 1990,
3. Using your previous two answers, find in what year the amount of healthy life lost to tobacco was expected to first equal that lost of diarrhea.

Group 2

Application to Business
A cable television company estimates that with 4 thousand subscribers its monthly revenue and cost (in thousands of dollars) are

$$R(x) = 32x - 0.21x^2 \tag{1}$$

$$C(x) = 195 + 12x \tag{2}$$

Determine the company's break-even points; that is, find the number of subscribers at which the revenue equals the cost.

Group 3

Application to Life Sciences
Salmonella bacteria, found on almost all chicken and eggs, grow rapidly in a nice warm place. If just a few hundred bacteria are left on the cutting board when a chicken is cut up, and they get into the potato salad, the population begins compounding. Suppose the number present in the potato salad after t hours is given by

$$f(t) = 500 \cdot 2^{3t}$$

1. If the potato salad is left out on the table, how many bacteria are present 1 hour later?
2. How many were present initially?
3. How often do the bacteria double?
4. How quickly will the number of bacteria increase to 32,000?

Group 4

Solve the following equation.

$$2^{x^2-4x} = \left(\frac{1}{16}\right)^{x-4}$$

Group 5

Application to Business and Economics
Lauren Snowden puts $10,500 into an account to save money to buy a car in 12 years. She expects the car of her dreams to cast $35,000 by then. Find interest rate that is necessary if the interest is computed using the following methods.

1. Compounded quarterly
2. Compounded monthly

Extra Problems

1. **General Interest (Art History)**
   Antonio Gaudi was a well-known architect from Spain (1852-1926). He became famous by creating a rather unique style combining interest in history, mathematics and nature. One of the most common geometrical figures used by Gaudi are known

to be the *Catenary Arcs* present in Casa Batllo, La Pedrera (Casa Mila) and the cathedral La Sagrada Familia. For example, the following is a picture of the attic of Casa Batllo:

Figure 1: Attic Casa Batllo

Assume that the biggest arc in this attic can be roughly approximated by the quadratic polynomial $-50x^2 + 12x + 24$. Could you determine which is the largest distance from the floor to the roof?

2. Graph each parabola and give its vertex, axis of symmetry, x-intercepts, and y-intercept.

   (a) $y = x^2 + 5x + 6$,
   (b) $y = \frac{1}{3}x^2 - \frac{8}{3}x + \frac{1}{3}$,

3. **Application to Public Health**
   From 1992 to 2011, the age adjusted incidence rate of invasive lung and bronchial cancer among women can be closely approximated by

$$f(t) = -0.0335t^2 + 0.490t + 48.4 \tag{3}$$

   where t is the number of years since 1992. Based on this model, in what year did the incidence rate reach a maximum? On what years was the rate increasing? Decreasing?

4. **Application to Business and Economics**
   Find the interest earned on $10,000 invested for 5 years at 4% interest compounded as follows.

(a) Annually

(b) Semiannually (twice a year)

(c) Quarterly

(d) Monthly

(e) Continuously

5. Graph the following.

$$y = -3e^{-2x} + 2$$